Available online at www.sciencedirect.com
SCIENCE DIRECT®

PERGAMON

International Journal of Heat and Mass Transfer 46 (2003) 2149–2154

International Journal of HEAT and MASS TRANSFER

www.elsevier.com/locate/ijhmt

Role of the contact layer between liquid and solid on a solidification process

Zygmunt Lipnicki *

University of Zielona Góra, Zielona Gora 65-426, Poland

Received 7 October 2002

Abstract

The object of this paper is the presentation of a theoretical solidification model of heat-conducting liquid metal flowing near of a cold plate. The problem of the unsteady behaviour both of the frozen metal layer and of the cold plate is studied analytically. Influence of the contact layer between the frozen layer and the cold plate on the solidification process is also studied. The results in the form of the analytical formula and the graphs are presented and compared with the other results.

1. Introduction

The paper examines analytically the solidification of a heat-conducting liquid metal flowing in the vicinity of a cooling plane. Solidification of flowing liquid metal being in contact with the cold plate is very important from the practical point of view. The thickness of the frozen layer metal is especially important in technology production of the printed circuit electronics. Generally the production of such materials consists of covering the tin layer of the cold copper plate. There is the contact layer between the frozen layer and the cold plate which role in the heat transfer will be also of the object of this paper. The present study will be help to find out about creation in the time of the frozen layer thickness on the cold plate.

Among other things in the works [1–4,6,7] the problem of solidification of flowing liquid on the cold plane wall was examined. The papers concern the cases of an isothermal cold plane wall and a cold wall of negligible heat capacity. Transient solidification of a flowing liquid on the wall theoretically was studied by Epstein [3]. In this work the problem of the unsteady behaviour of the frozen layer in the liquid flowing past the cold plane wall was studied analytically. In the paper [4] the solidification of flowing liquid on the cold plane wall with the contact layer between the cold plane and frozen layer was studied. Authors of the paper [4] used the very simple theoretical model to solve the problem. They assumed the constant temperature in the all area of the cold plate. The temperature depends only on time.

In this paper temperature in the cold plate depends also on location. The way of calculation of the temperature on the cold plane was presented.

2. Presentation of the problem

A photo of the contact layer between both the cold plate (copper plate) and the frozen layer (tin crust), which in the solidification process was arisen is shown in Fig. 1. In this place the solution of continuity (there is lack of continuity) occurs, that is the form of the solid solution of one metal in the other. The structure of this layer is composed. A non-pure substance occupies this volume. The gaps of air can also be present. There is the additional resistance of the flow of heat from the heater place to the cooler place. The knowledge of the resistance of heat in the contact layer is very important from the practical point of view and the new theoretical and experimental investigations are needed.

The flowing liquid metal (see Fig. 2) is not overheated and has the temperature T_F, which is the fusion temperature of the material. Very important among other things are both the thickness of the frozen layer δ_1, which is

* Fax: +48-3247290.
E-mail address: z.lipnicki@iis.uz.zgora.pl (Z. Lipnicki).

0017-9310/03/$ - see front matter
doi:10.1016/S0017-9310(02)00531-8

Nomenclature

λ_s, c_s, ρ_s	thermal conductivity, specific heat and density of liquid metal
λ_c, c_c, ρ_c	thermal conductivity, specific heat and density of cold plate
a_s, a_c	thermal diffusivity of both liquid metal and cold plate
L	latent heat of liquid metal fusion
H	cold plate thickness
T_F	liquid metal fusing point
T_∞	temperature of liquid metal far from frozen layer surface
T_0	initial temperature of cold plate
α	heat transfer coefficient between liquid metal and crust
α_{CON}	liquid metal-cold plate contact layer heat transfer coefficient
T	temperature
T_W	temperature on plane of cold plate
δ_1	frozen layer thickness
x_1	co-ordinate
t	time

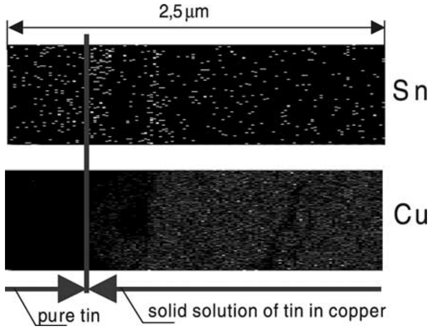

Fig. 1. Contact layer in view of X-ray.

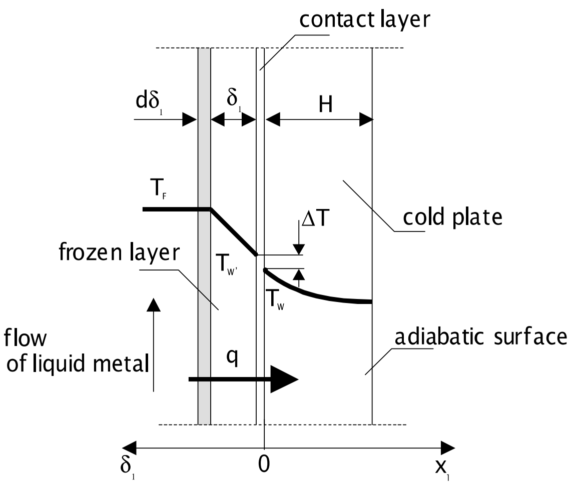

Fig. 2. Theoretical model.

developing on the plate, and the velocity increases of the frozen layer thickness $d\delta_1/dt$. The main object of this paper is modifying of the temperature field T and of the layer thickness depended on the cooling conditions, which are determined by the properties of both the cold plate and liquid metal. The cooling plane of the cold plate has the temperature T_W, which depends on time t. Between the crust and the copper plate there is the contact layer, which causes, the additional resistance of the flowing heat. In the contact layer the different temperature $\Delta T = T_{W'} - T_W$ arises, where $T_{W'}$ is the temperature of the plane of the frozen layer of the opposite cold plate. The other plane of the cold plate is adiabatic.

The result of the solidification process, the front of solidification (solidification interface) translocates from the boundary surface of the cold plate to outside with the velocity $d\delta_1/dt$. Liquid–solid phase-change heat transfer phenomena (solidification) are accompanied by release of thermal energy in this place. The stream heat q released in the front of the interface is transferred through the frozen layer to the cold plate. The cold plate absorbs all the released heat.

By virtue of the assumption the solidification front is sharp and planer. We assume that the change of the accumulation heat of the frozen layer is very small in comparison with the cold plate. All thermodynamics parameters are considered constant.

The energy balance with equates the instantaneous latent heat and from the liquid flowing metal by convection to the frozen layer may be described as:

$$\rho_s L \frac{d\delta_1}{dt} + \alpha(T_\infty - T_F) = \lambda_s \frac{T_F - T_{W'}}{\delta_1};$$
$$\lambda_s \frac{T_F - T_{W'}}{\delta_1} = \alpha_{CON}(T_{W'} - T_W) \tag{1}$$

from above we received

$$T_{W'} = T_W + \rho_s \frac{L}{\alpha_{CON}} \cdot \frac{d\delta_1}{dt} + (T_\infty - T_F)\frac{\alpha}{\alpha_{CON}}.$$

The following set of dimensionless parameters was introduced: dimensionless time, frozen layer thickness, co-ordinate, temperature and wall temperature

$$\tau = Fo \cdot Ste, \quad \delta = \frac{\delta_1}{H}, \quad x = \frac{x_1}{H}, \quad \theta = \frac{T - T_0}{T_F - T_0},$$
$$\theta_W = \frac{T_W - T_0}{T_F - T_0},$$

where are the Fourier number, the Stefan number and the Biot number

$$Fo = \frac{t \cdot a_s}{H^2}; \quad Ste = \frac{c_s \cdot (T_F - T_0)}{L}; \quad Bi = \frac{H \cdot \alpha_{CON}}{\lambda_s}$$

and another dimensionless parameters

$$a = \frac{a_c}{a_s}; \quad \lambda = \frac{\lambda_c}{\lambda_s}.$$

In view of the fact that the liquid flowing metal is not overheated, from this assumption, the heat does not flow to the cold plate by convection. For the reason the Eqs. (1) in dimensionless form may be written

$$1 - \theta_W = \frac{\delta \cdot d\delta}{d\tau} + \frac{1}{Bi}\frac{d\delta}{d\tau}. \tag{2}$$

Analysis of the temperature on the boundary of the plate requires the special attention. The knowledge of the temperature $\theta_W(\tau)$ is necessary to solve the above Eq. (2). It is necessary also, to define the boundary conditions between the plate and the crust liquid. This condition will searched in further analysis.

The differential equation of conduction of heat in the cold plate has the form:

$$\frac{\partial\theta}{\partial\tau} = \frac{a}{Ste}\frac{\partial^2\theta}{\partial x^2}, \tag{3}$$

which allows to find the temperature distribution θ [5]. For the case, which is interesting for us, the first boundary of the plate (see Fig. 2) $x = 0$ is kept at temperature θ_W, the second boundary $x = 1$ is isolated $(\partial\theta/\partial x = 0)$ and the initial temperature is $\theta(x, 0) = 0$. The total expression for both the boundary and the initial conditions may be written as:

$$\theta(x,0) = 0, \quad \frac{\partial\theta}{\partial x}(1,\tau) = 0, \quad \theta(0,\tau) = \theta_W. \tag{4}$$

Eqs. (2)–(4) create the system of differential equations, which should be solved together.

3. Solution of the problem

We introduce the boundary condition on the plane of the cold plate in the form

$$\theta_W = 1 - \exp(-\beta\tau), \tag{5}$$

with the unknown constant β, which will be determined by the following analysis. Substituting this value into Eq. (2) after transformation we get

$$\theta_W = 1 - \exp(-\beta\cdot\tau) \Rightarrow \exp(-\beta\cdot\tau) = 1 - \theta_W$$
$$= \frac{\delta \cdot d\delta}{d\tau} + \frac{1}{Bi}\frac{d\delta}{d\tau}. \tag{6}$$

This formula from $\tau = 0$ to ∞ will be integrated what allow to receive parameter β (in this case of the frozen layer thickness changes from $\delta = 0$ to δ_{max})

$$\int_0^\tau \exp(-\beta\tau)\cdot d\tau = \frac{\delta^2}{2} + \frac{\delta}{Bi}$$
$$\Rightarrow \int_0^\infty \exp(-\beta\tau)\cdot d\tau = \frac{\delta^2_{max}}{2} + \frac{\delta_{max}}{Bi}$$
$$\Rightarrow \beta = \frac{1}{\frac{\delta^2_{max}}{2} + \frac{\delta_{max}}{Bi}}. \tag{7}$$

The use of the global energy balance for the cold plate lead to the equation of the maximum layer thickness δ_{max}:

$$\rho_s \cdot L \cdot H \cdot \delta_{max} = c_c \cdot \rho_c \cdot (T_F - T_0) \cdot H$$
$$\Rightarrow \delta_{max} = \frac{\lambda \cdot Ste}{a} \tag{8}$$

and then to the constant β

$$\beta = \frac{1}{\frac{\delta^2_{max}}{2} + \frac{\delta_{max}}{Bi}} = \frac{1}{\frac{1}{2}\left(\frac{\lambda\cdot Ste}{a}\right)^2 + \frac{1}{Bi}\left(\frac{\lambda\cdot Ste}{a}\right)}. \tag{9}$$

The equation used for the calculation of the frozen layer thickness is:

- when the Biot number is equal $Bi = \infty$ (for example: when is connection of the tin and of the copper, from (9) the constant parameter $\beta = 0.625$ was received) from Eq. (6) we get the formula of the layer thickness

$$\frac{\delta^2}{2} = \frac{1}{\beta}\cdot(1 - \exp(-\beta\cdot\tau))$$
$$\Rightarrow \delta = \sqrt{\frac{2}{\beta}\cdot(1 - \exp(-\beta\cdot\tau))}; \tag{10}$$

- when the Biot number has values $Bi < \infty$ (see Eq. (6)) the frozen layer may be calculated from the formula

$$\delta = -\frac{1}{Bi} + \sqrt{\frac{1}{Bi^2} + \frac{2}{\beta}\cdot(1 - \exp(-\beta\cdot\tau))}. \tag{11}$$

It is required to find the solution of the equation of conduction of heat (3) in the finite region $0 < x < 1$ with the initial and boundary conditions (4), at time $\tau > 0$. In order to solve Eq. (3) the temperature is given by

$$\theta(x,\tau) = 1 - e^{-\beta\cdot\tau} + \psi(x,\tau), \tag{12}$$

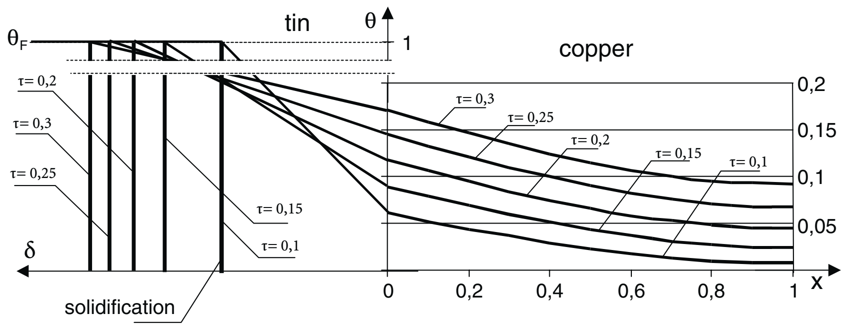

Fig. 3. Effect of time, τ, on frozen layer thickness, δ, and temperature distribution, θ, for perfect between cold plate and solidification metal.

after transformation, we get a new equation

$$\frac{\partial\psi}{\partial\tau} = \frac{a}{Ste}\frac{\partial^2\psi}{\partial x^2} - \beta \cdot e^{-\beta\cdot\tau} \tag{13}$$

with both the new initially and boundary conditions

$$\psi(0,\tau) = 0 \quad \text{and} \quad \psi(x,0) = 0, \quad \frac{\partial\psi}{\partial x}(1,\tau) = 0. \tag{14}$$

We postulate a solution of Eq. (13) in the form

$$\psi(x,\tau) = \sum_{n=1}^{\infty} A_n \cdot \left(e^{-\frac{a}{Ste}\cdot k_n^2\cdot\tau} - e^{-\beta\cdot\tau}\right) \cdot \sin(k_n \cdot x). \tag{15}$$

We have now to see whether this function also satisfies the boundary and initial conditions. How we can easily see the above equation, the initial condition has fulfillment, it is clear that the second boundary condition will be fulfilled for every value of the parameter

$$k_n = \frac{\pi}{2} + \pi \cdot (n-1).$$

A constant A_n we will find from the following equation

$$\sum_{n=1}^{\infty} A_n \cdot \left(\beta - k_n^2 \cdot \frac{a}{Ste}\right) \cdot \sin(k_n \cdot x) = -\beta,$$

which was received from Eqs. (13) and (15). Hence, if we assume the possibility of the expansion and that we may integrate term by term, we have

$$A_n \cdot \left(\beta - k_n^2 \cdot \frac{a}{Ste}\right) \cdot \int_0^1 \sin^2(k_n \cdot x) \cdot dx = -\beta \cdot \int_0^1 \sin(k_n x) \cdot dx$$

and

$$A_n = \frac{2\cdot\beta}{k_n \cdot \left(k_n^2 \cdot \frac{a}{Ste} - \beta\right)}.$$

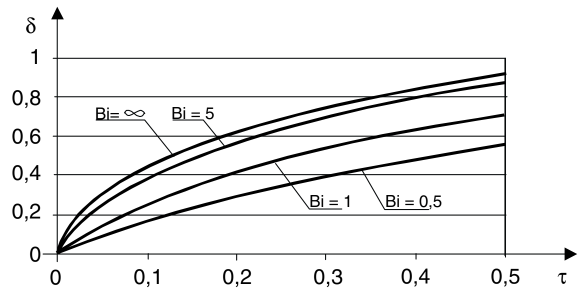

Fig. 4. Frozen layer thickness for different Biot numbers for contact layer tin/copper.

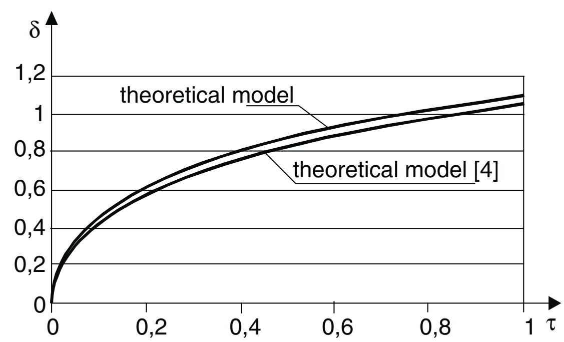

Fig. 5. Compared frozen layer thickness for perfect contact layer ($Bi = \infty$) tin/copper.

Finally, an exact analytical solution of Eq. (3) can be expressed as

$$\theta(x,\tau) = 1 - e^{-\beta\cdot\tau} + 2\cdot\beta\sum_{n=1}^{\infty}\frac{1}{k_n}\frac{1}{\frac{a}{Ste}k_n^2 - \beta} \times \left[e^{-\frac{a}{Ste}\cdot k_n^2\cdot\tau} - e^{-\beta\cdot\tau}\right]\sin(k_n \cdot x). \tag{16}$$

The solutions of Eqs. (7), (8), (10) and (16) and the graphical form are presented in Figs. 3–8.

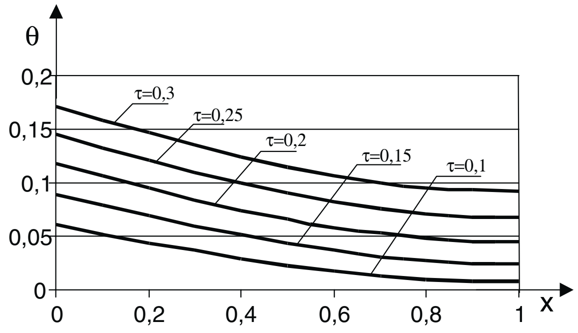

Fig. 6. Temperature distributions in copper plate for perfect contact layer tin/copper.

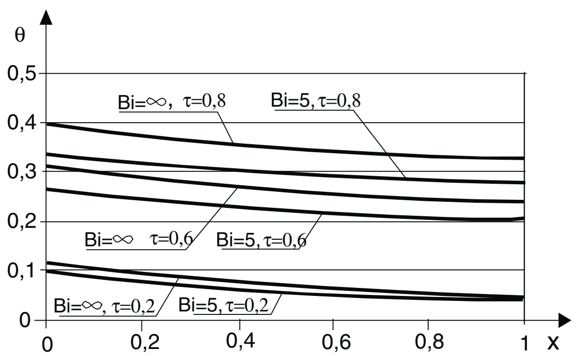

Fig. 7. Compared distribution temperature in copper plate for both different Biot numbers.

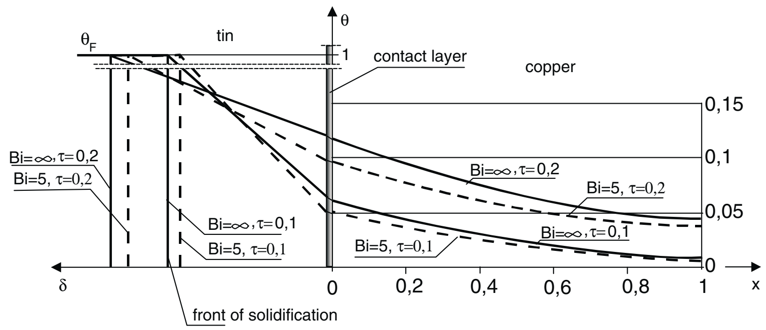

Fig. 8. Compared distribution of temperature for different Biot numbers.

Fig. 3 shows the location of the front solidification and the temperature distribution in these layers for the perfect contact (see also Fig. 6). The distribution of the temperature is monotonically for the different time.

Influence of the resistance heat of the contact layer on the frozen layer developed is evident, if the Biot number is bigger, the thickness of the frozen layer grows up faster with time (see Fig. 4). Great influence we can see especially for the small Biot number then the solidification process is very slow. Comparison of two methods is presented in Fig. 5. We can see that compression of the frozen layer thickness calculated on the basis of the simple theoretical model with the exact theoretical model, which is presented here, reveals good agreement. Figs. 5 and 8 show a comparison of the predicted theoretical distribution of the temperature profiles for the two different conditions.

4. Results comparison and conclusion

The theoretical model, which describes analytically of unsteady behaviors of the frozen layer in liquid metal flowing near the cold plate, was presented. It describes behavior of both the dimensionless of the frozen layer, which is created on the cold plane and the area of the temperature in the plate (Figs. 3–8). The theoretical model depends on three parameters ($\lambda Ste/a$, a/Ste, Bi). The first, which is equal with the maximum thickness frozen layer (δ_{max}) and the second parameter represent thermodynamically properties of two metals, which appear on the heat contact solidification surface: liquid–solid. These parameters are very important in the considered phenomenon and very convenient in use. The third parameter, Biot number (Bi) determines properties of the contact layer, which depends on thermal diffusivity in this layer. There is the forth-unknown parameter (β), which is determined here. Thanks to making all the analysis, the conditions on the plane of the cold plate were found.

It is easy to show that the results received from equation are in agreement with the simulation method Calcosoft-2D, it was early shown in work [4]. The heat resistance between the cold plate and the frozen layer in the solidification process plays very important role (see: Figs. 4 and 7).

References

[1] C. Lapadula, W.K. Mueller, Heat conduction with solidification and convection boundary condition at the

freezing front, Int. J. Heat Mass Transfer 9 (1966) 702–705.

[2] J.M. Saviano, R. Siegel, An analytical solution for solidification of a moving warm liquid onto an isothermal cold wall, Int. J. Heat Mass Transfer 12 (1969) 803–809.

[3] M. Epstein, The growth and decay of a frozen layer in forced flow, Int. J. Heat Mass Transfer 19 (1976) 1281–1288.

[4] Z. Ignaszak, Z. Lipnicki, A. Bydalek, Solidification of liquid metal flowing below a cold Plate, Fifth International Pamir Conference on Fundamental and Applied MHD, vol. 2, II131–II136, Ramatuelle, France, 2002.

[5] H.S. Carslaw, J.C. Jaeger, Conduction of Heat in Solid, Oxford University Press, 1959.

[6] W. Longa, Krzepnięcie odlewów, Wydawnictwo Śląsk, Katowice, 1985.

[7] R. Viskanta, Phase-change heat transfer, in: G.A. Lane (Ed.), Solar Heat Storage: Latent Heat Materials, CRC Press, Int., Bocaratod, FL, 1983, pp. 153–221 (Chapter 5).